

an upper bound of  $S$ ;  
 for if  $y$  be any number less than  $G$ ,  
 $\exists$  at least one member  $y$  of  $S$   
 which is greater than  $y$ .  
 Thus,  $G$  is the least of all the upper  
 bounds of  $S$  i.e.  $\sup S = G$ .

### (10) Illustrations:

1. If a real number  $k$  is not an upper bound  
 of a set  $S$ , then  $\exists$  at least one elt  
 $y \in S$  such that  $k < y$ .  
 $k$  upper bound of  $S$   $\Rightarrow$   $\exists$  no  $y \in S$  such that  $k < y$

(2) If a real number  $k$  is not a lower  
 bound of a set  $S$ , then  $\exists$  at least  
 one element  $z \in S$  such that  $z < k$ .

(3) If a set has one upper bound, then  
 it has an infinite number of upper bounds

$\rightarrow$  Notice that,  
 if  $k$  is an upper bound of a set  $S$ , then  
 any number  $k_1 (> k)$  is also an upper  
 bound of  $S$ .

(4) If a set has one lower bound of a  
~~set~~  $S$ , then it has an infinite number  
 of lower bounds.

$\rightarrow$  Notice that if  $k$  is a lower bound of  
 a set  $S$ , then any number  $k_1 (< k)$  is  
 also a lower bound of  $S$ .